ECE 320
Energy Conversion and Power Electronics
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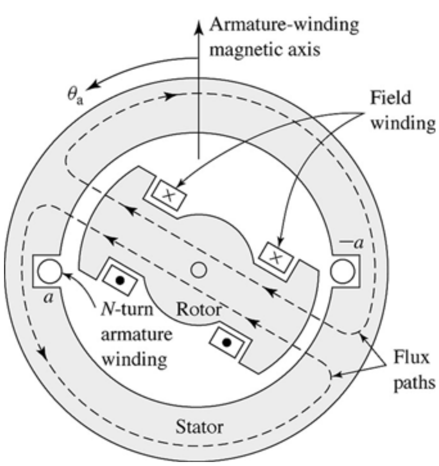
Chapter 5: Three Phase Windings
(Textbook Sections 4.3-4.7)

Chapter Objectives

Flux linkage plays a crucial role in the operation of both DC and AC machines. In this chapter, the geometry and the operation of windings in AC machines is discussed. The flux varies in time, and can also vary in position, or be stationary. To understand how these machines operate, the concept of *space vectors* (or *space phasors*) is introduced.

5.1 Introduction

Electric machines often have defined an armature winding which is the winding that is power producing, and a field winding that generates the magnetic field. Either could be on the stator or rotor depending on the specific motor or generator; however it is more common with AC machines such as synchronous or induction machines that the armature winding is on the stator (the stationary portion of the motor). Synchronous machines have field windings on the rotor that is excited by direct current delivered to the rotor windings by slip rings or collector rings by carbon brushes. The field winding produces the north and south poles, thus the image shown in Figure 1 is for a two-pole, single phase (one armature winding) synchronous generator. The magnetic axis for the armature winding is perpendicular to the area defined by the armature winding (armature winding is the perimeter of this area).



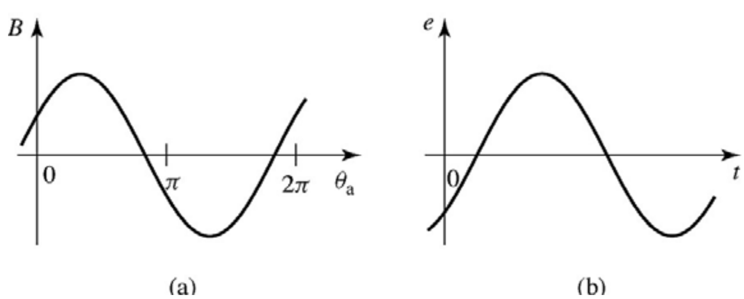


Figure 1. Figures 4.4 and 4.5 from your textbook showing a simple two-pole, single phase synchronous generator, the spatial distribution of the magnetic field relative to the magnetic axis of the armature winding, and the time dependent induced voltage in the armature winding [1].

In a three phase device, the armature has three coils each with a magnetic axis that is rotated spatially by 120º as shown in Figure 2.

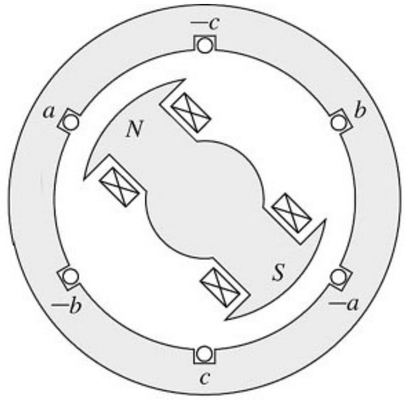


Figure 2. A three-phase, two-pole synchronous generator as shown in Figure 4.12(a) in your textbook [1].

More poles for the field winding and more armature windings are also possible as shown in Figure 3. Such a configuration can deliver three phase power by interconnecting the armature windings in a Y connection configuration as an example.

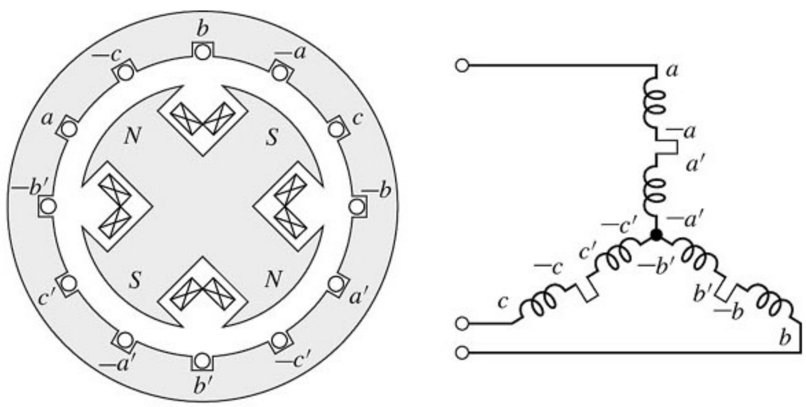


Figure 3. A four-pole synchronous generator with multiple armature windings that can be wired together in a three-phase Y connection is shown (from Figure 4.12(b) and 4.12(c) in your textbook [1]).

5.2 Control of the Magnetomotive Force Distribution

As generators, the synchronous machines typically use the stator windings as a source of electrical power. As motors, the stator (or armature) windings are commonly supplied electrical power to generate a spatially varying field (we will consider the time variation of these fields later). Insight to the distribution (in space) of the field from the armature windings can be seen by considering a single N-turn coil on the stator that spans 180 electrical degrees (in 180º it has gone from a +F to a –F). Such a coil is known as a *full pitch coil*. The mmf distribution for such a full pitch coil is depicted in Figure 4.19 from your textbook as shown below in Figure 4.

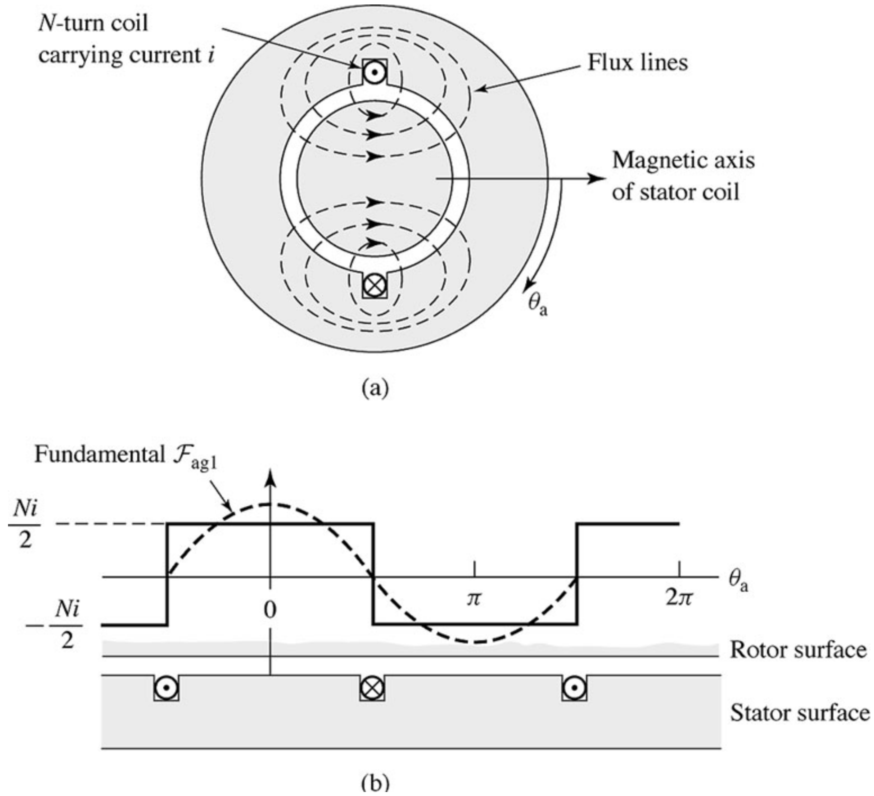


Figure 4. A full pitch coil on the stator. Windings on the rotor are left off for clarity. This is Figure 4.19 in your textbook. (a) Schematic view of flux produced by a concentrated, full-pitch winding in a machine with a uniform air gap. (b) The air-gap mmf produced by current in this winding [1].

Assuming the reluctances of the stator and rotor negligible compared to the gaps, then the full mmf of $N\cdot i$ would drop across two gaps as the flux traverses a full loop. This gives rise to a $+(N\cdot i/2)$ maximum, and a $-(N\cdot i/2)$ minimum for the mmf as one spatially maps F for the stator winding as a function of the angle relative to its magnetic axis. Plotting F as a function of this angle, θ_a, shows abrupt changes at the wires of the coil. The Fourier transform of this square wave gives a fundamental sinusoidal distribution of F_{ag1} as shown in Figure 4.

Harmonics exist since it is non-sinusoidal. In an attempt to make the spatial distribution more sinusoidal in form, multiple coils can be placed within specific groves of the stator as shown for one of the phases in a three-phase winding in Figure 5. Here there are equal numbers of wires in each of the groves, and equal current in each of the wires.

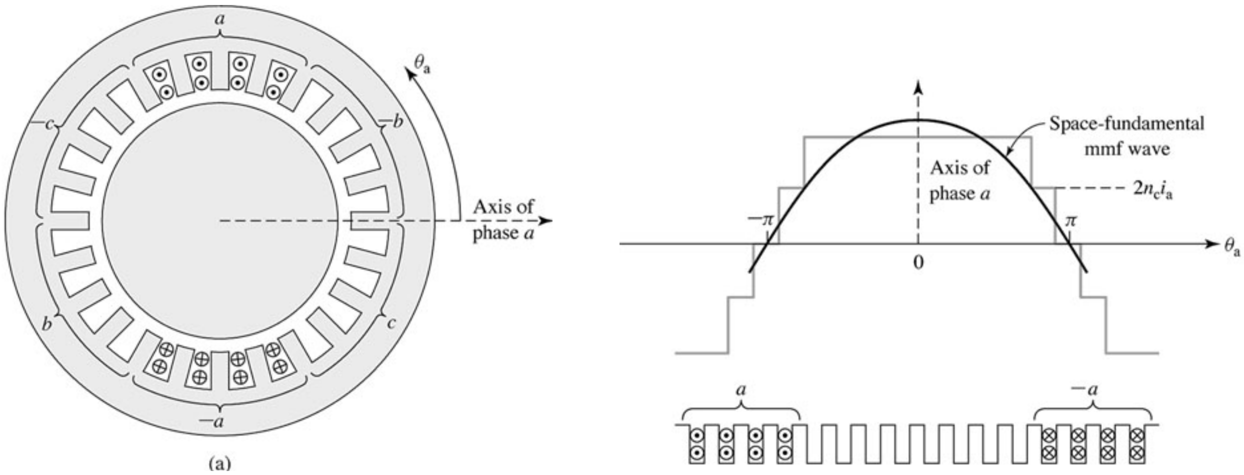


Figure 5. The mmf of one phase of a distributed two-pole, three phase winding with full-pitch coils. This is image 4.20 from your textbook [1].

By adjusting the number of turns in each slot a more sinusoidal mmf distribution can be also be obtained - as shown for windings of a rotor in the configuration shown in Figure 6.

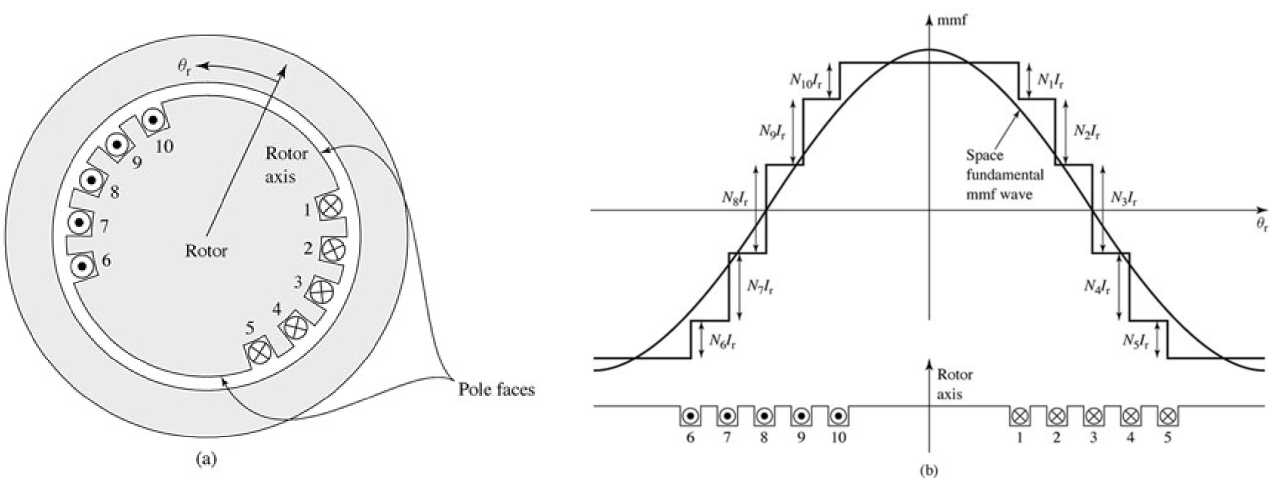


Figure 6. The air-gap mmf of a distributed winding on the rotor of a round-rotor generator. This is image 4.21 from your textbook [1].

Some simple methods for controlling the shape of the magnetomotive force for a set of coils have been outlined. Now we turn our attention to how we use this information for analyzing the electric machine through the concept of space vectors.

5.3 Current Space Vectors

Consider three identical windings placed in a space of uniform permeability as shown in Figure 7. Each winding carries a time dependent current, $i_1(t)$, $i_2(t)$, and $i_3(t)$, We also require that

$$i_1(t) + i_2(t) + i_3(t) \equiv 0 \qquad \textbf{(5.1)}$$

Each current produces a flux in the direction of the coil axis, and if we assume the magnetic medium to be linear, we can find the total flux by adding the individual fluxes. This means that we could produce the same flux by having only one coil, identical to the three, but placed in the direction of the total flux, carrying an appropriate current. If the coils in Figure 7 carry the following currents i_1 = 5 (A), i_2 = -8 (A), i_3 = 3 (A), then the vectoral sum of these currents, oriented in the same direction as the corresponding coil can be shown in Figure 8.

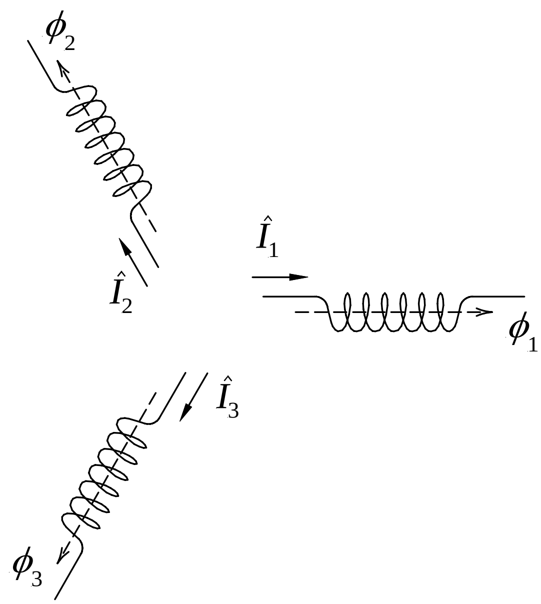


Figure 7. Three phase windings spatially oriented 120º apart.

The direction of the resultant coil and current it should carry, we create three vectors, each in the direction of one coil, and equal in amplitude to the current of the coil it represents. If, for example, the coils are placed at angles of 0º, 120º, 240º. Then their vectoral sum will be:

$$\mathbf{i} = i\,\underline{/\phi} = i_1 + i_2 e^{j120^\circ} + i_3 e^{j240^\circ} \qquad \textbf{(5.2)}$$

This represents the vector of a current oriented in space, and thus we define **i** as a space vector. If i_1, i_2, and i_3 are functions of time, so will be the amplitude and angle of **i**. If we consider a horizontal axis in space as a real axis, and a vertical axis as an imaginary axis, then we can find the real (i_d = Re{**i**}) and imaginary (i_q = Im{**i**}) components of the space vector. With this representation, we can determine the three currents from the space vector as:

$$
\begin{aligned}
i_1(t) &= \frac{2}{3}\mathrm{Re}\{\mathbf{i}(t)\} \\
i_2(t) &= \frac{2}{3}\mathrm{Re}\{\mathbf{i}(t)e^{-j\gamma}\} \\
i_3(t) &= \frac{2}{3}\mathrm{Re}\{\mathbf{i}(t)e^{-j2\gamma}\}
\end{aligned}
\qquad \textbf{(5.3)}
$$

$$
\gamma = 120^\circ = \frac{2\pi}{3}\,\mathrm{rad} \qquad \textbf{(5.4)}
$$

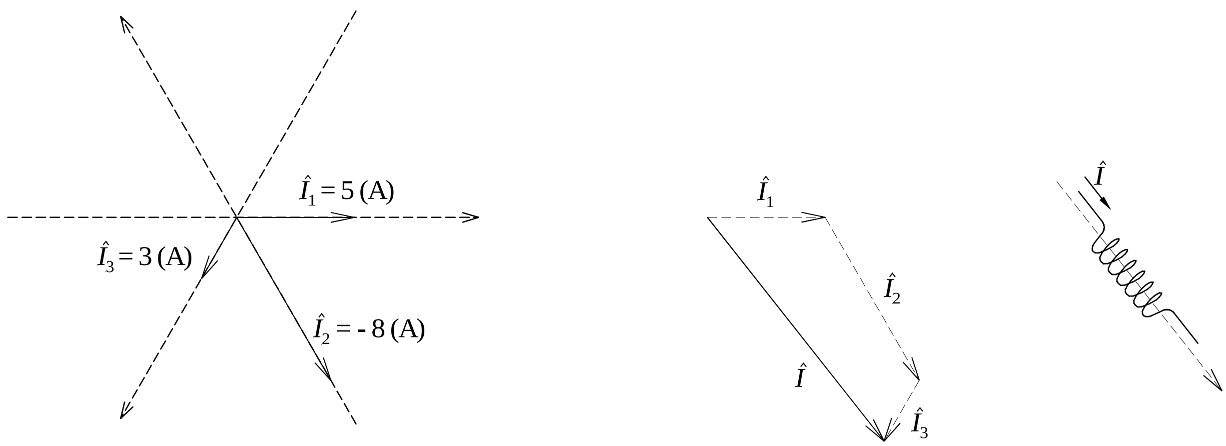
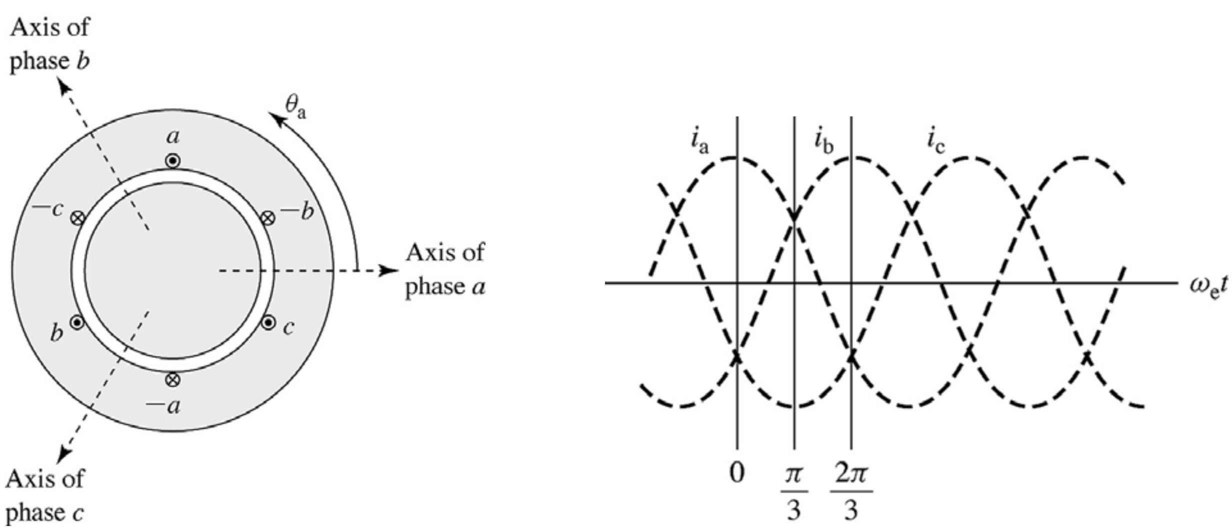


Figure 8. (a) Currents in the three windings of Figure 7. (b) Resultant space vector and (c) corresponding winding position and current of an equivalent single coil.

Consider if the three coils in Figure 7 were to represent the three stator windings of an AC machine as shown in Figure 9, with currents for each phase also shown in Figure 9.



(a) (b)

Figure 9. Simplified two pole 3-phase stator winding and the instantaneous currents for each phase. These are Figures 4.29 and 4.30 from your textbook [1].

Then for:

$$
\begin{aligned}
i_a(t) &= I_m \cos(\omega_e t) \\
i_b(t) &= I_m \cos(\omega_e t - 120^\circ) \\
i_c(t) &= I_m \cos(\omega_e t + 120^\circ)
\end{aligned} \tag{5.5}
$$

the space vector is:

$$
\begin{aligned}
\mathbf{i}(t) &= I_m\left[\cos(\omega_e t) + e^{j120^\circ}\cos(\omega_e t - 120^\circ) + e^{j240^\circ}\cos(\omega_e t + 120^\circ)\right] \\
&= I_m\left[\cos(\omega_e t) + \{\cos(120^\circ) + j\sin(120^\circ)\}\cos(\omega_e t - 120^\circ) + \{\cos(240^\circ) + j\sin(240^\circ)\}\cos(\omega_e t + 120^\circ)\right] \\
&= I_m\left[\cos(\omega_e t) - 0.5\cos(\omega_e t - 120^\circ) - 0.5\cos(\omega_e t + 120^\circ) + j\frac{\sqrt{3}}{2}\{\cos(\omega_e t - 120^\circ) - \cos(\omega_e t + 120^\circ)\}\right]
\end{aligned} \tag{5.6}
$$

Using: $\cos(x \pm y) = \cos(x)\cos(y) \mp \sin(x)\sin(y)$

$$
\begin{aligned}
\mathbf{i}(t) &= I_m\left[\cos(\omega_e t) - 0.5\left\{-0.5\cos(\omega_e t) + \frac{\sqrt{3}}{2}\sin(\omega_e t)\right\} - 0.5\left\{-0.5\cos(\omega_e t) - \frac{\sqrt{3}}{2}\sin(\omega_e t)\right\}\right] \\
&\quad + jI_m\frac{\sqrt{3}}{2}\left[-0.5\cos(\omega_e t) + \frac{\sqrt{3}}{2}\sin(\omega_e t) + 0.5\cos(\omega_e t) + \frac{\sqrt{3}}{2}\sin(\omega_e t)\right] \\
&= \frac{3}{2}I_m\left[\cos(\omega_e t) - j\sin(\omega_e t)\right]
\end{aligned} \tag{5.7}
$$

This is a space vector that rotates in space as a function time at an angular frequency of ω_e. Graphically, this can be seen from summation of i_a, i_b, and i_c in Figure 9 (b) at different points in time ($\omega_e t$ = 0, $\pi/3$, and $2\pi/3$), the resultant space vector at each point in time is represented by the corresponding magnetomotive force, **F**, associated with that space vector as shown in Figure 10.

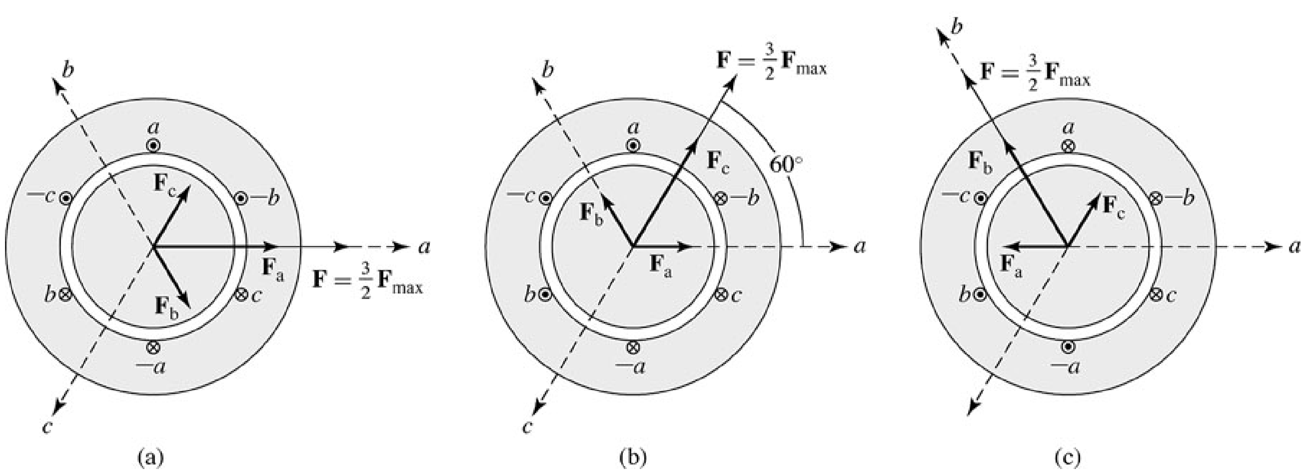


Figure 10. The production of a rotating magnetic field by means of three-phase currents.

Section 5.2 in these notes described ways of forming magnetomotive force spatial distributions that were more sinusoidal in profile. Representing such windings as sinusoidally concentrated windings in the stator as depicted in Figure 11, then the density of turns of a the coil would vary sinusoidally in space as function of the angle θ.

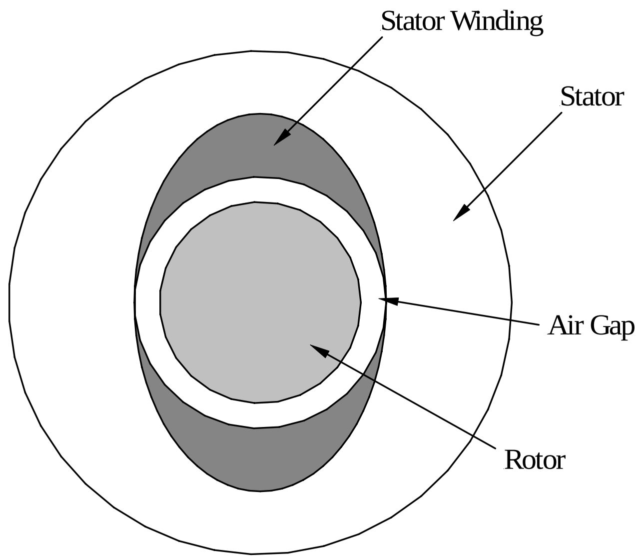


Figure 11. The production of a rotating magnetic field by means of three-phase currents.

Thus the number of turns, dN_s, covering an angle $d\theta$ at a position θ over $d\theta$ is a sinusoidal function of the angle θ. The turns density, $n_{s1}(\theta)$ is then:

$$\frac{dN_s}{d\theta} = n_{s1}(\theta) = \hat{n}_s \sin\theta \tag{5.8}$$

For the total number of turns, N_s, in the winding:

$$N_s = \int_0^{\pi} n_{s1}(\theta) d\theta \tag{5.9}$$

This leads to

$$n_{s1}(\theta) = \frac{N_s}{2} \sin\theta \tag{5.10}$$

With i_1 current flowing through this winding, the flux density in the air gap between the rotor and the stator can be found for the integration path shown in Figure 12. The path of integration is defined by the angle q and we can notice that because of symmetry of the flux density at the two air gap segments in the path is the same.

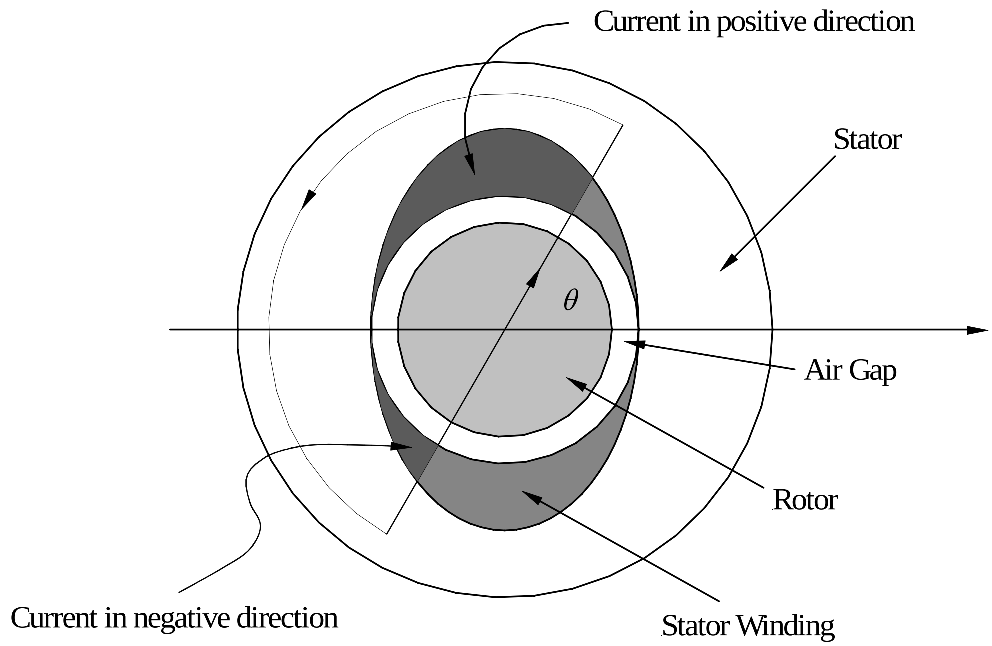


Figure 12. Integration path to calculate flux density in the air gap.

Assuming the permeability of the stator and rotor is infinite, then $H_{iron} = 0$ and:

$$
\begin{aligned}
2H_{g1}(\theta)g &= \int_{\theta}^{\theta+\pi} i_1 n_{s1}(\phi)d\phi \\
\frac{2B_{g1}(\theta)}{\mu_0}g &= i_1 N_s \cos\theta \\
B_{g1}(\theta) &= i_1 \frac{N_s \mu_0}{2g}\cos\theta
\end{aligned}
\qquad \textbf{(5.11)}
$$

For a given current, i_1, in the coil the flux density in the air gap varies sinusoidally with angle, but as shown in Figure 13 it reaches a maximum when the angle θ is zero. For the same machine and conditions as in Figure 13, Figure 14 shows the plot of turns density, $n_s(\theta)$ and flux density, $B_g(\theta)$ in cartesian coordinates with θ as the horizontal axis. For this one coil, if the current, i_1, were to vary sinusoidally in time, then the flux density would also change in time. The direction of the space vector would be maintained however the amplitude would change in time. The nodes of the flux density where it is equal to zero will remain at 90º and 270º, while the extrema of the flux will be maintained at 0º and 180º.

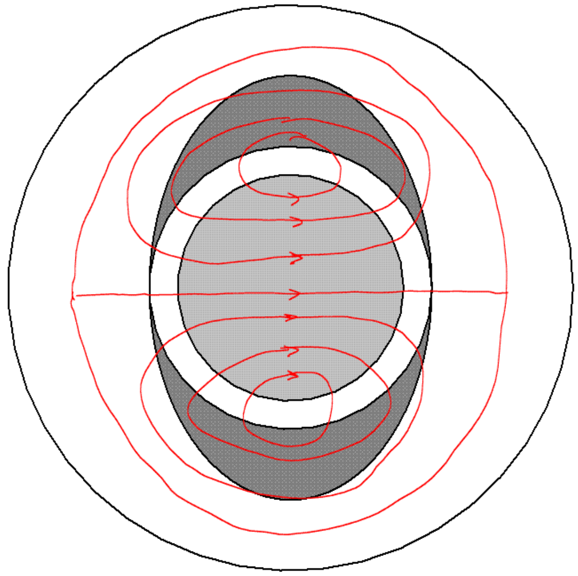


Figure 13. Sketch of the flux (red) in the air gap.

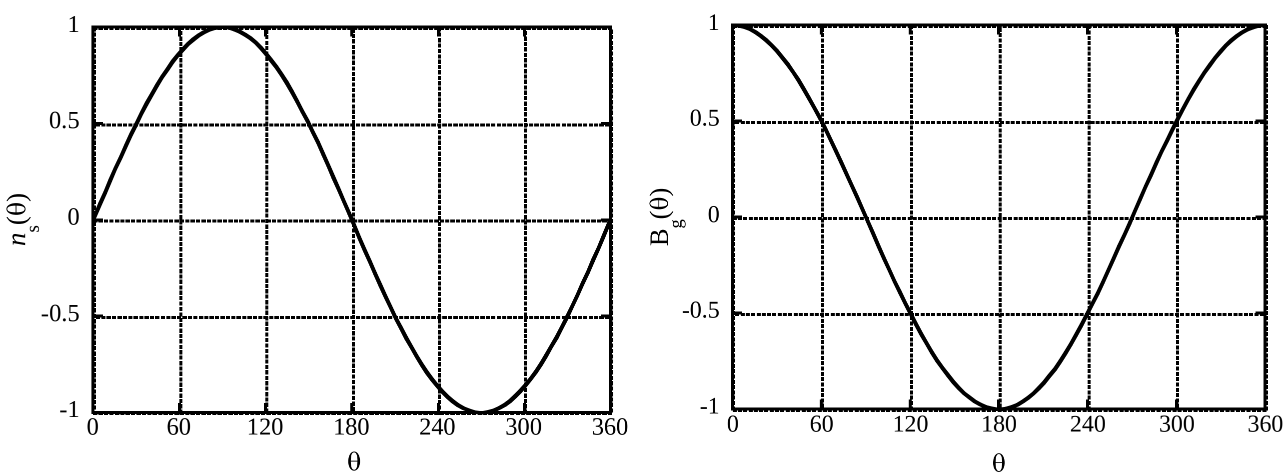


Figure 14. Turns density on the stator and the air gap flux density vs. θ.

Consider now an additional winding, identical to the first, but rotated with respect to it by 120º. For a current in this winding we will get a similar air gap flux density as before, but with nodes at 210º = 90º + 120º and at 30º = 270º + 120º. If a current, i_2, is flowing in this winding, then the air gap flux density due to it will follow a form similar to equation (5.11) but rotated by 120º = (2π/3).

$$B_{g2}(\theta) = i_2 \frac{N_s \mu_0}{2g} \cos\left(\theta - \frac{2\pi}{3}\right) \qquad \textbf{(5.12)}$$

Similarly, a third winding, rotated 240º relative to the first winding and carrying current i_3, will produce an air gap flux density of:

$$B_{g3}(\theta) = i_3 \frac{N_s \mu_0}{2g} \cos\left(\theta - \frac{4\pi}{3}\right) \qquad \textbf{(5.13)}$$

Combining these three flux densities, we obtain a sinusoidally distributed air gap flux density, that could equivalently come from a winding placed at an angle ϕ and carrying current i as:

$$B_g(\theta) = B_{g1}(\theta) + B_{g2}(\theta) + B_{g3}(\theta) = i\,\frac{N_s \mu_0}{2g}\cos(\theta + \phi) \tag{5.14}$$

This means that as the currents change, the flux could be due instead to only one sinusoidally distributed winding with the same number of turns. The location, $\phi(t)$, and current, $i(t)$, of this winding can be determined from the current space vector:

$$\mathbf{i}(t) = i(t)\underline{/\phi} = i_1(t) + i_2(t)e^{j120^\circ} + i_3(t)e^{j240^\circ} \tag{5.15}$$

5.3.1 Balanced, Symmetric Three-phase Currents

If the currents i_1, i_2, i_3 form a balanced three-phase system of frequency $f_s = \omega_s/2\pi$, then we can write:

$$\begin{aligned}
i_1 &= \sqrt{2}I\cos(\omega_s t + \phi_1) = \frac{\sqrt{2}}{2}\left[\mathbf{I}_s e^{j\omega_s t} + \mathbf{I}_s e^{-j\omega_s t}\right] \\
i_2 &= \sqrt{2}I\cos\left(\omega_s t + \phi_1 - \frac{2\pi}{3}\right) = \frac{\sqrt{2}}{2}\left[\mathbf{I}_s e^{j(\omega_s t - 2\pi/3)} + \mathbf{I}_s e^{-j(\omega_s t - 2\pi/3)}\right] \\
i_3 &= \sqrt{2}I\cos\left(\omega_s t + \phi_1 - \frac{4\pi}{3}\right) = \frac{\sqrt{2}}{2}\left[\mathbf{I}_s e^{j(\omega_s t - 4\pi/3)} + \mathbf{I}_s e^{-j(\omega_s t - 4\pi/3)}\right]
\end{aligned} \tag{5.16}$$

where $\mathbf{I}$ is the phasor corresponding to the current in phase 1. The resultant space vector is:

$$\mathbf{i}_s(t) = \frac{3}{2}\frac{\sqrt{2}}{2}\mathbf{I}e^{j\omega_s t} = \frac{3}{2}\frac{\sqrt{2}}{2}Ie^{j(\omega_s t + \phi_1)} \tag{5.17}$$

The resultant flux density wave is then:

$$B(\theta, t) = \frac{3}{2}\sqrt{2}I\,\frac{N_s \mu_0}{2g}\cos(\omega_s t + \phi_1 - \theta) \tag{5.18}$$

which shows a travelling wave, with a maximum value of $B_{\max} = \frac{3}{2}\sqrt{2}I\,\frac{N_s}{\mu_0}$. This wave travels around the stator at a constant speed ω_s, as shown Figure 15.

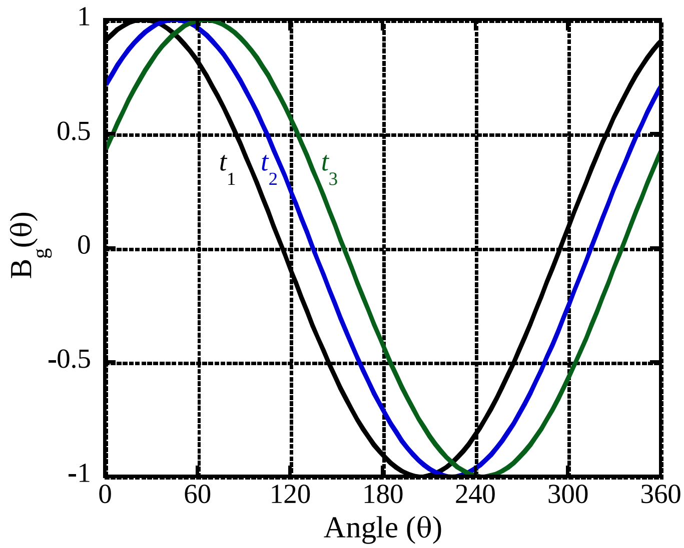


Figure 15. Air gap flux density profile vs. θ for three times $t_3>t_2>t_1$.

5.4 Phasors and Space Vectors

This is a good point to reflect on the differences between phasors and space vectors. A current phasor, $\hat{I} = Ie^{j\phi_0}$, describes *one* sinusoidally varying current of frequency ω, amplitude $\sqrt{2}I$, and initial phase ϕ_0. The sinusoid can be reconstructed from the phasor as:

$$i(t) = \frac{\sqrt{2}}{2}\left[\mathbf{I}e^{j\omega t} + \mathbf{I}^* e^{-j\omega t}\right] = \sqrt{2}I\cos(\omega t + \phi_0) = \mathrm{Re}\left(\mathbf{I}e^{j\omega t}\right) \qquad \textbf{(5.19)}$$

Although rotation is implicit in the definition of the phasor, no rotation is described by it.

On the other hand, the definition of current space vector requires *three* currents that sum to zero. These currents are implicitly in windings that are symmetrically placed, but the currents are not necessarily sinusoidal. Generally the amplitude and angle of the space vector changes with time, but no specific pattern is *a priori* defined. We can reconstruct the three currents that constitute the space vector from equation (5.3). When these constituent currents form a balanced, symmetric system, of frequency ω_s, then the resultant space vector is of constant amplitude, rotating at a constant speed. In that case, the relationship between the phasor of one current and the space vector is shown in equation (5.17).

5.5 Magnetizing Current, Flux, and Voltage

To see how this rotating magnetic flux influences the windings we use Faraday's law. From here on we will use sinusoidal symmetric three-phase quantities.

The three stationary windings are linked by a rotating flux as shown in Figure 16. When the current is maximum in phase 1, the flux is as shown in Figure 16(a) and is linking all of the turns in phase 1. Later, the flux has rotated as show in Figure 16(b) then the flux linkages with coil 1 have decreased. When the flux has rotated 90º as in Figure 16(c), the flux linkages with the phase 1 windings are zero.

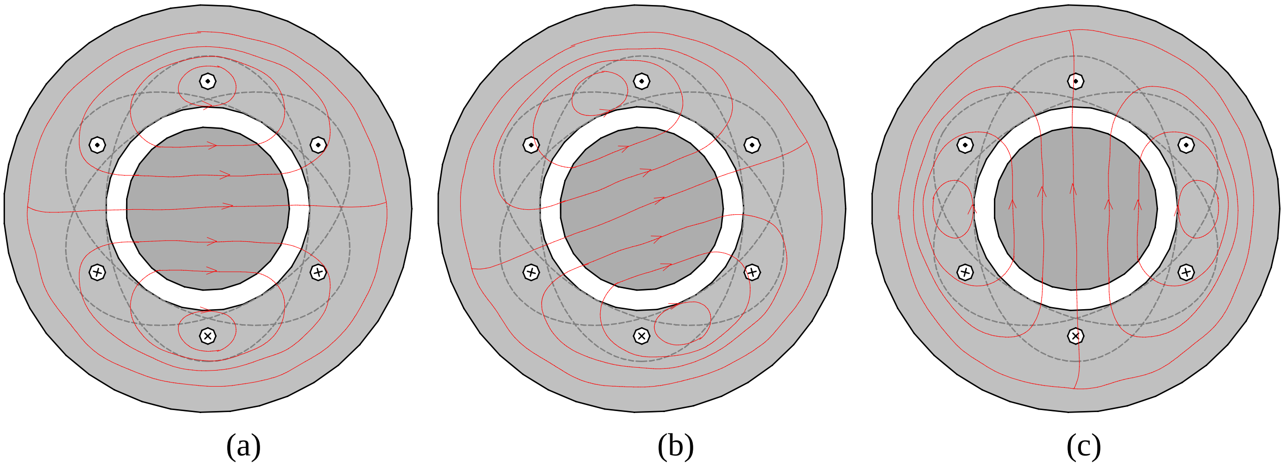


Figure 16. Rotating flux and flux linkages. Sinusoidal windings containing many turns in the stator are represented by single wires to show current flow direction.

To calculate the flux linkages, l, we determine the flux through a differential number of turns at an angle θ as shown in Figure 17.

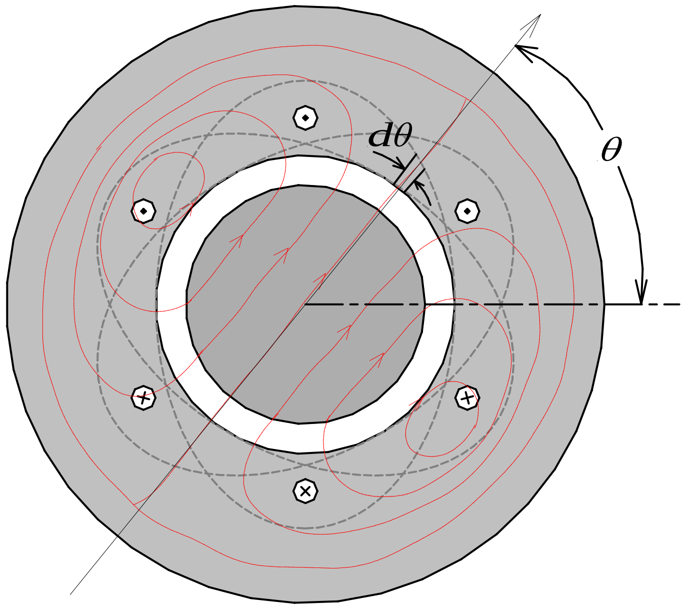


Figure 17. Flux linkage through a differential number of turns (such as one turn) for coil 1.

The flux through this differential section of coil is then:

$$\phi(t,\theta) = \int_{\theta-\pi}^{\theta} B_g(t,\theta) dA = lr \int_{\theta-\pi}^{\theta} B_g(t,\theta) d\theta \qquad \textbf{(5.20)}$$

where l is the axial length of the coil (into the page), and r is the radius to the coil.

The number of turns linked by this flux is $dn_s(\theta) = n_s(\theta)d\theta$, so the flux linkages for these few (or one) turns is:

$$d\lambda = n_s(\theta)d\theta \cdot \phi(\theta) \tag{5.21}$$

To find the flux linkages, λ_1, for all of coil 1, then we must integrate the flux linkages over all turns of coil 1 or:

$$\lambda_1 = \int d\lambda = \int_0^{\pi} n_s(\theta)d\theta \cdot \phi(\theta) \tag{5.22}$$

When these two integrals are taken, then we find:

$$\lambda_1(t) = \frac{N_s^2 lr}{8g} 3\pi\mu_0 \sqrt{2} I \cos(\omega_s t + \phi_1) = L_M \sqrt{2} I \cos(\omega_s t + \phi_1) \tag{5.23}$$

which means the flux linkages in coil 1 are in phase with the current in this coil and proportional to it. The flux linkages of the other two coils, 2 and 3, are identical to that of coil 1, but lagging in time by 120º and 240º respectively. With these three quantities we can create a flux-linkage space vector, $\boldsymbol{\lambda}$ as:

$$\boldsymbol{\lambda} = \lambda_1 + \lambda_2 e^{j120^\circ} + \lambda_3 e^{j240^\circ} = L_M \mathbf{i} \tag{5.24}$$

Since the flux linkages of each coil vary, and in our case sinusoidally, a voltage is induced in each of these coils. The induced voltage in each coil is 90º ahead of the current in it, bringing to mind the relationship of current and voltage of the inductor. Notice though, that it is not just the current in the winding that causes the flux linkages and the induced voltages, but rather the current in all three windings. Although this is the case, we sill call the constant L_M the magnetizing inductance, and the induced voltages in each coil can be found as:

$$\begin{aligned} e_1(t) &= \frac{d\lambda_1}{dt} = \omega\sqrt{2} I \cos\left(\omega t + \phi_1 + \frac{\pi}{2}\right) \\ e_2(t) &= \frac{d\lambda_2}{dt} = \omega\sqrt{2} I \cos\left(\omega t + \phi_2 + \frac{\pi}{2} - \frac{2\pi}{3}\right) \\ e_3(t) &= \frac{d\lambda_3}{dt} = \omega\sqrt{2} I \cos\left(\omega t + \phi_3 + \frac{\pi}{2} - \frac{4\pi}{3}\right) \end{aligned} \tag{5.25}$$

and the voltage space vector, **e** can be defined as:

$$\mathbf{e} = e_1 + e_2 e^{j120^\circ} + e_3 e^{j240^\circ} = j\omega L_M \mathbf{i} \tag{5.26}$$

The flux linkage space vector is aligned with the current space vector, while the voltage space vector, e, is ahead of both by 90º. This agrees with the fact that the individual phase voltages lead the current by 90º, as shown in

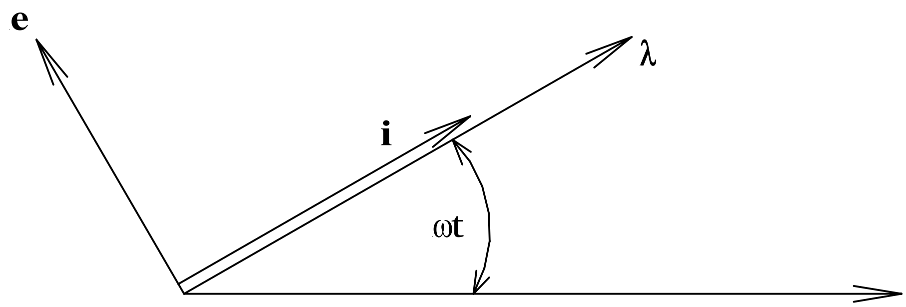


Figure 18. Magnetizing current, flux-linkage, and induced voltage space vectors.

1 A. E. Fitzgerald, C. Kingsley, Jr., S. D. Umans, *Electric Machinery, 6th edition*, McGraw-Hill, New York, 2003.